Math 270 Day 6 Part 2

Section 2.6: Substitutions and Transformations

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What we'll go over in this section

- **The Substitution/Transformation to use to solve a Homogeneous Equation**
- **The Substitution/Transformation to use to solve the Equation $\frac{dy}{dx} = G(ax + by)$**
- The Substitution/Transformation to use to solve a Bernoulli Equation
- The Substitution/Transformation to use to solve Equations with Linear Coefficients

Section 2.6: Substitutions and Transformations

Intro

- We've already learned how to solve separable, first-order, and exact equations
- In this section we will look at 4 types of differential equations that can be transformed to an equation that we already know how to solve

Substitution Procedure

(a) Identify the type of equation and determine the appropriate substitution or transformation.
(b) Rewrite the original equation in terms of new variables.
(c) Solve the transformed equation.
(d) Express the solution in terms of the original variables.

Section 2.6: Substitutions and Transformations

The Substitution/Transformation to use to solve a Homogeneous Equation

Homogeneous Equation

Definition 4. If the right-hand side of the equation

$$\frac{dy}{dx} = f(x, y) \tag{1}$$

can be expressed as a function of the ratio y/x alone, then we say the equation is **homogeneous.**

For example, the equation $(x - y)\,dx + x\,dy = 0$

can be written in the form $\frac{dy}{dx} = \frac{y - x}{x} = \frac{y}{x} - 1$.

The equation $(x - 2y + 1)\,dx + (x - y)\,dy = 0$

can be written in the form $\frac{dy}{dx} = \frac{x - 2y + 1}{y - x} = \frac{1 - 2(y/x) + (1/x)}{(y/x) - 1}$

and cannot be expressed as a function of y/x alone

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The Substitution/Transformation to use to solve a Homogeneous Equation

Homogeneous Equation

Definition 4. If the right-hand side of the equation

$$(1) \qquad \frac{dy}{dx} = f(x, y)$$

can be expressed as a function of the ratio y/x alone, then we say the equation is **homogeneous.**

To solve a homogeneous equation

1) Make the substitution $v = \frac{y}{x}$
2) You'll end up with a separable DE

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The Substitution/Transformation to use to solve a Homogeneous Equation

Example 1 Solve $(xy + y^2 + x^2)\,dx - x^2\,dy = 0$.

Section 2.6: Substitutions and Transformations

The Substitution/Transformation to use to solve the Equation $\frac{dy}{dx} = G(ax + by)$

Equations of the Form $\frac{dy}{dx} = G(ax + by)$ **What does that mean?**

To solve $\frac{dy}{dx} = G(ax + by)$

1) Make the substitution $z = ax + by$
2) You'll end up with a separable DE

Section 2.6: Substitutions and Transformations

The Substitution/Transformation to use to solve the Equation $\frac{dy}{dx} = G(ax + by)$

Example 2 Solve $\frac{dy}{dx} = y - x - 1 + (x - y + 2)^{-1}$.